Economics 6003 Quantitative Economics

Identification in VARs: Beyond Contemporaneous Restrictions

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Beyond Contemporaneous Restrictions ...

- In the last lectures, we have considered identification in VARs via contemporaneous exclusion restrictions – setting some elements of the matrix mapping the reduced form shocks to the structural shocks to zero:
  - Recursive ID $\rightarrow$ there is a Wold causal ordering which enables us to derive a unique mapping. This corresponds to imposing a triangular, Cholesky factorization upon the reduced form variance/covariance matrix.
  - Contemporaneous, structural ID $\rightarrow$ a generalization of the recursive ID approach where we require that the reduced form variance/covariance factorization satisfy the rank condition, to ensure that we have a unique mapping.
- In response to concerns about the validity of contemporaneous exclusion restrictions, the applied literature has explored alternative structural restrictions for identification.

Beyond Contemporaneous Restrictions ...

- We'll take a look at three recent strategies in this vein, with some discussion of their applications.
    - restrictions on the long-run effects of structural shocks
    - restrictions on the signs of the structural impulse responses at certain horizons
    - restrictions on the variance/covariance structure *over time*
- All of these strategies maintain the same baseline, linear, dynamic structure for the reduced form VAR. Recall our bivariate, VAR(1) system:

$$x_t = A_0 + A_1 x_{t-1} + e_t$$

We'll now consider how the identification strategies impose structure on the reduced form relationships in this baseline system.

Long-run Restrictions

- Blanchard and Quah (1989) are credited with the idea of using long-run restrictions to identify a VAR. They considered a bivariate system of the change in log output and the unemployment rate, which they thought of as mapping into a Keynesian-style AS-AD framework (despite no price measure). Thus, there are aggregate supply and demand shocks that have a structural interpretation in their model. How to isolate them?
- Based upon Phelps's work, the structural restriction that they imposed for identification was:
  - aggregate demand shocks have no long-run (permanent) effect upon the log output level.

  With this restriction, they can back-out the structural shocks from the reduced form system. Let's take a look.

Long-run Restrictions

- Assume that the VAR has an SVAR representation, where:

$$
\begin{aligned}
&Bx_t = \Gamma_0 + \Gamma_1 x_{t-1} + \varepsilon_t \Rightarrow \\
&A_0 = B^{-1}\Gamma_0,\ A_1 = B^{-1}\Gamma_1 \text{ and} \\
&e_t = B^{-1}\varepsilon_t, E\left(e_t e_t'\right) = \Omega \Rightarrow \Phi = B\Omega B'
\end{aligned}
$$

- This implies the following VMA representation:

$$
x_t = \bar{A}_0 + \sum_{h=0}^{\infty} A_1^h B^{-1} \varepsilon_{t-h}
$$

where $\bar{A}_0 = A_0 \left(\sum_{h=0}^{\infty} A_1^h\right)$.

Long-run Restrictions

- The long-run effect on the log output level is defined to be the *cumulated* impulse response at an infinite horizon to the shocks. So, we have:

$$\begin{aligned} \frac{dx_{\infty}}{d\varepsilon_t} &= \sum_{h=0}^{\infty} A_1^h B^{-1} \\ &= \left(\sum_{h=0}^{\infty} A_1^h \right) B^{-1} \\ &= \widetilde{A_1} B^{-1} \end{aligned}$$

  where $\widetilde{A_1}$ is the 2×2 matrix resulting after all the multiplying and summing.

- Writing the matrices out, we have:

$$\widetilde{A_1} B^{-1} = \begin{bmatrix} \tilde{a}_{11} & \tilde{a}_{12} \\ \tilde{a}_{21} & \tilde{a}_{22} \end{bmatrix} \begin{bmatrix} 1 & -b_{12} \\ -b_{21} & 1 \end{bmatrix} \left(\frac{1}{1 - b_{12} b_{21}} \right)$$

Long-run Restrictions

- Simplifying further, we have:

$$\Rightarrow \begin{bmatrix} (\tilde{a}_{11} - \tilde{a}_{12} b_{12}) & (-\tilde{a}_{11} b_{12} + \tilde{a}_{12}) \\ (\tilde{a}_{21} - \tilde{a}_{22} b_{21}) & (-\tilde{a}_{21} b_{12} + \tilde{a}_{22}) \end{bmatrix} \left(\frac{1}{1 - b_{12} b_{21}} \right)$$

- Let the first variable in the system be denoted y and be the change in log output. Let the second variable in the system be denoted z and be the unemployment rate. Assume that the structural shock for unemployment is the demand shock.
- The restriction that the long-run effect of ε_z is zero (no permanent level output effect from a demand shock) is equivalent to the restriction:

$$\begin{aligned} (-\tilde{a}_{11} b_{12} + \tilde{a}_{12}) &= 0 \\ \Rightarrow b_{12} &= \frac{\tilde{a}_{12}}{\tilde{a}_{11}} \end{aligned}$$

Long-run Restrictions

- So, we're making some headway – we have a value for the contemporaneous effect of a supply shock on the unemployment rate that is only given in terms of observables (the A_1 matrix is estimable from the reduced form).
- We can go farther by leveraging the information in the variance/covariance matrix:

$$
\begin{aligned}
var\left(\varepsilon_t\right) &= \begin{bmatrix} \sigma_y^2 & 0 \\ 0 & \sigma_z^2 \end{bmatrix} \\
&= B\Omega B' \\
&= \begin{bmatrix} 1 & b_{12} \\ b_{21} & 1 \end{bmatrix} \begin{bmatrix} \sigma_1^2 & \sigma_{12} \\ \sigma_{12} & \sigma_2^2 \end{bmatrix} \begin{bmatrix} 1 & b_{21} \\ b_{12} & 1 \end{bmatrix}
\end{aligned}
$$

- Notice how $b_{12} \neq b_{21}$ and $\sigma_{12} = \sigma_{21}$, since the B matrix is structural while Ω is reduced form.

Long-run Restrictions

- Simplifying further, we have:

$$\Rightarrow \begin{bmatrix} \left(\sigma_1^2 + \sigma_{12} b_{12}\right) & \left(\sigma_{12} + \sigma_2^2 b_{12}\right) \\ \left(\sigma_1^2 b_{21} + \sigma_{12}\right) & \left(\sigma_{12} b_{21} + \sigma_2^2\right) \end{bmatrix} \begin{bmatrix} 1 & b_{21} \\ b_{12} & 1 \end{bmatrix}$$

$$= \begin{bmatrix} \left(\sigma_1^2 + 2\sigma_{12} b_{12} + \sigma_2^2 b_{12}^2\right) & \left(\sigma_1^2 b_{21} + \sigma_{12}\left(1 + b_{12} b_{21}\right) + \sigma_2^2 b_{12}\right) \\ \left(\sigma_1^2 b_{21} + \sigma_{12}\left(1 + b_{21} b_{12}\right) + \sigma_2^2 b_{12}\right) & \left(\sigma_1^2 b_{21} b_{21} + 2\sigma_{12} b_{21} + \sigma_2^2\right) \end{bmatrix}$$

$$\Rightarrow \begin{array}{c} \left(\sigma_1^2 + 2\sigma_{12} b_{12} + \sigma_2^2 b_{12}^2\right) = \sigma_y^2 \\ \left(\sigma_1^2 b_{21} b_{21} + 2\sigma_{12} b_{21} + \sigma_2^2\right) = \sigma_z^2 \\ \left(\sigma_1^2 b_{21} + \sigma_{12}\left(1 + b_{21} b_{12}\right) + \sigma_2^2 b_{12}\right) = 0 \end{array}$$

which are 3 equations in 3 unknowns $(b_{21}, \sigma_y^2, \sigma_z^2)$. Thus, we can identify all of the structural parameters from observables with the long-run restriction.

Long-run Restrictions – IV Interpretation

- In this simple VAR(1), there is a nice IV interpretation.
  - The long-run restriction on the effect on the output level means that the effect of once-lagged unemployment upon the change in log output is the negative of the contemporaneous effect of unemployment upon the change in log output.
  - Imposing this, we can then use the lagged change in log output and the lagged unemployment rate as instruments for the lagged change in log output and the change in the unemployment rate in the output equation.
  - We can then take the residuals from the output equation and use them as instruments along with lagged change in log output and the lagged unemployment rate for contemporaneous output and lagged output and unemployment in the unemployment equation.
- This should work fine, so long as the instruments are strong enough (we'll talk about this more later).

Long-run Restrictions

- Based upon their identification (in richer VAR - more lags), Blanchard and Quah (1989) found:
  - A demand shock leads to hump-shaped responses of output and unemployment.
    - Output rises and peaks at about 4 quarters. It then declines until zero after about 5 years (never negative).
    - Unemployment falls and bottoms out at about 4 quarters. It then gradually rises towards zero, hitting it at about 5 years (never positive).
  - A supply shock (e.g., a surprise fall in oil prices or the like) leads to somewhat strange results.
    - Output rises, peaking at about 8 quarters and 8 times the initial positive effect. There is a curious move down of output at 1 quarter, but then a steady move up. In the long-run, output stays permanently higher (how is this possible?).
    - Unemployment *rises* for about 3 quarters and then falls. It goes negative at 5 quarters, bottoming out at about 10 quarters, and then converging to zero.

Blanchard-Quah (1989) demand shock IRFs

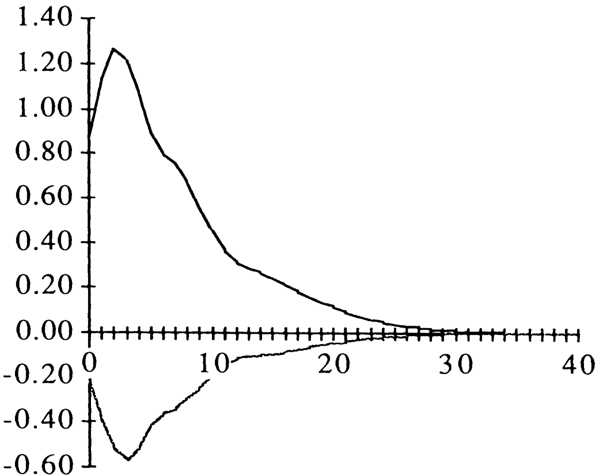


FIGURE 1. RESPONSE TO DEMAND, — = OUTPUT, ~~~ = UNEMPLOYMENT

Blanchard-Quah (1989) supply shock IRFs

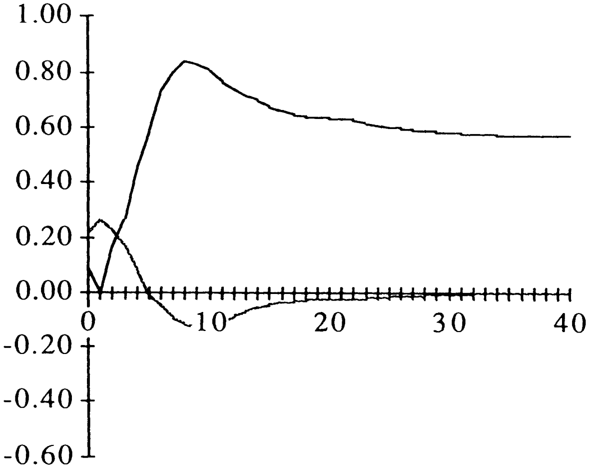


FIGURE 2. RESPONSE TO SUPPLY, — = OUTPUT, ~~~ = UNEMPLOYMENT

Long-run Restrictions

- Difficulties with this approach?
    - Faust and Leeper (1997) found that long-run restrictions do a poor job when considering the sample sizes (length of samples) usually considered (30-40 years).
    - If there is a lot of persistence in the variables, then it may be that we need longer samples to get an accurate sense of the long-run response.
    - Fry and Pagan (2005) note that the situation is worse. Even in long samples, things may behave badly.
    - They note that the approach here has an IV interpretation. Recent work on IV has shown that if the instruments are only *weakly* correlated with the endogenous variables, then the IV estimates can be really bad and unreliable.
    - Fry and Pagan show that weak instruments in the SVAR with long-run restrictions is equivalent to high persistence in the series with the long-run restricted shock. This does not go away with longer samples.
    - With many macroeconomic time series, there is a lot of persistence.

Long-run Restrictions

- Does this mean that the approach is useless?
- Not necessarily. It does mean that we need to be careful in applying long-run restrictions approaches.
- We might augment the usual approach with an evaluation of the strength of the implicit instruments, using some of the recent work on weak instruments.
- Fry and Pagan (2005) effectively do this by looking at the persistence of the unemployment rate for the Blanchard and Quah model. They find that it is highly persistent, which means that the BQ results are probably unreliable.
- There has been a lot of work on how to construct inference that is robust to weak instruments. This may supply a means of recovering some of the usefulness of the long-run restrictions. I don't know of any applied work that does this yet, but it would be interesting.

Sign Restrictions

- In scientific work, it desirable to look for the weakest set of assumptions under which a conclusion can be derived. VAR identification based upon *sign restrictions* is based upon this criterion.
- The basic idea is to generate myriad impulse response functions from myriad identification schemes, which each generate a set of *orthogonal* shocks. Then, only keep those IRFs which fulfill the sign restrictions. Examples are Faust (1998) and his *partial* identification framework, and Uhlig (1997, 2005) and his investigation of U.S. monetary policy.
    - For example, a monetary policy contraction cannot raise prices for 6 months. Thus, any IRF from the set of identification schemes which *does* show a positive response of prices to an interest rate rise are discarded.
- Sign restrictions are weak assumptions – they do not identify a unique structural model.

Sign Restrictions

- Consider our baseline VAR/SVAR. Recall the mapping from the reduced form to the structural shocks is:

$$e_t = B^{-1}\varepsilon_t$$

- Notice how we can easily rework and relabel this to get an alternative expression that has an identical reduced form:

$$\begin{aligned} e_t &= B^{-1}S^{-1}S\varepsilon_t \\ &= T\eta_t, \text{ where } T = B^{-1}S^{-1} \end{aligned}$$

  Here, the entries in the diagonal matrix S are the reciprocals of the associated standard deviations of the structural shocks ε.
- Thus, η is uncorrelated across rows and has unit variances.

Sign Restrictions

- The observational equivalence between $T\eta$ and $B^{-1}\varepsilon$ hints at a method to generate a host of observationally equivalent structural models (viz., models with orthogonal shocks).
- Suppose that we have an orthogonal matrix:

$$\begin{aligned} Q \text{ orthogonal} &\Rightarrow Q'Q = QQ' = I \Rightarrow \\ e_t &= TQ'Q\eta_t \\ &= \tilde{T}\tilde{\eta}_t \end{aligned}$$

where $\tilde{T} = TQ'$ and $\tilde{\eta} = Q\eta$. The number of observationally equivalent identification schemes is only limited by the number of conformable orthogonal matrices:

$$\begin{aligned} E\left(\eta_t\eta_t'\right) &= I \Rightarrow \\ E\left(\tilde{\eta}_t\tilde{\eta}_t'\right) &= QE\left(\eta_t\eta_t'\right)Q' = QQ' = I \end{aligned}$$

Sign Restrictions

- Let's consider a particular class of orthogonal matrices known as *Givens rotation* matrices. They use the trigonometric identity $\cos^2\theta + \sin^2\theta = 1$ to generalize the usual contemporaneous exclusion restrictions while preserving orthogonality of the shocks.
- In a bivariate system, it has the form:

$$Q = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}, \theta \in [0, \pi]$$

- Thus, we have an *infinite*-dimensional set of identification schemes indexed by θ (since it varies along a continuum). Notice how all of the resulting $\tilde{\eta}$ are indistinguishable based upon their first two moments. **However**, they will all have different dynamic effects since they will have different realized values.

Sign Restrictions

- How can we choose from this giant set of structural models?
  - Uhlig (1997,2005) and Faust (1998) argue that we can use economic theory to restrict ourselves to only those structural models which generate IRFs that fulfill a set of qualitative criteria, such as sign restrictions.
- How to summarize the properties of the subset of selected IRFs?
  - It is common to present information on the quantiles of the IRFs for each variable at each horizon. So, we have a *median* IRF from the set, with bounds defined by the *16.7%* and *83.3% quantiles*.
  - A problem noted by Fry and Pagan (2005) is that the median IRF constructed in such a manner actually mixes across a variety of structural models. In fact, there is no reason to think that the structural shocks across these different models are orthogonal. They propose that an alternative, unique median model IRF be presented.

How Sign Restrictions Work

- Fry and Pagan (2005) present a simple example of the partial identification strategy using sign restrictions. Suppose we have the following structural supply-demand system:

$$
\begin{aligned}
q_t &= -\beta p_t + \varepsilon_{d,t} \\
q_t &= \gamma p_t + \varepsilon_{s,t} \Rightarrow \\
p_t &= \frac{1}{\beta+\gamma}\left(\varepsilon_{d,t} - \varepsilon_{s,t}\right), \text{ in equilibrium.}
\end{aligned}
$$

- The sign restriction is that a demand shock raises the price, while a supply shock lowers the price. The shocks are orthogonal.
- Of course, if we want to estimate the system, we are confronted with the classic simultaneity problem. Are the sign restrictions sufficient to identify the contemporaneous effects of the shocks?

How Sign Restrictions Work

- We start the partial identification by picking an arbitrary identification that generates a set of orthogonal shocks – this is the fundamental property that we want to preserve. Consider the recursive structural system given by:

$$\begin{aligned} p_t &= \eta_{1,t} \\ q_t + \tau p_t &= \eta_{2,t} \Rightarrow \\ p_t &= \frac{1}{\tau}\left(\eta_{1,t} - \eta_{2,t}\right), \text{ in equilibrium.} \end{aligned}$$

- If $\tau > 0$, then notice how η_1 is like the demand shock, while η_2 is like the supply shock.
- If the recursive structural system is correct, then we can estimate τ via an OLS regression of q on $-p$.

How Sign Restrictions Work

The asymptotics of the OLS estimator of τ:

$$
\begin{aligned}
\hat{\tau} &\rightarrow -\frac{E\left(p_t q_t\right)}{E\left(p_t^2\right)} \\
&= -\frac{\left[\gamma E\left(p_t^2\right) + E\left(p_t \varepsilon_{s,t}\right)\right]}{E\left(p_t^2\right)} \\
&= -\gamma + \frac{\left(\frac{1}{\beta+\gamma}\right)\sigma_s^2}{\left(\frac{1}{\beta+\gamma}\right)^2\left(\sigma_d^2+\sigma_s^2\right)} \\
&= -\gamma + \frac{\left(\beta+\gamma\right)\sigma_s^2}{\left(\sigma_d^2+\sigma_s^2\right)} \\
&= \left(\frac{\sigma_d^2}{\sigma_d^2+\sigma_s^2}\right)\gamma + \left(\frac{\sigma_s^2}{\sigma_d^2+\sigma_s^2}\right)\beta \\
&\nrightarrow \frac{1}{\beta+\gamma}
\end{aligned}
$$

How Sign Restrictions Work

- So, we know that we definitely *won't* get the true contemporaneous impact using the recursive identification.
  - The sign is correct, since $\gamma, \beta > 0 \Rightarrow \tau > 0$.
  - However, the magnitude is all wrong.
- Regardless, η_1 and η_2 are orthogonal, by construction. We can start the generation of multiple IRFs via the Givens rotation applied to the baseline, recursively identified shocks

$$\begin{aligned}
\begin{bmatrix} \tilde{\eta}_{1,t} \\ \tilde{\eta}_{2,t} \end{bmatrix} &= \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix} \begin{bmatrix} \eta_{1,t} \\ \eta_{2,t} \end{bmatrix} \\
&= \begin{bmatrix} \eta_{1,t}\cos\theta - \eta_{2,t}\sin\theta \\ \eta_{1,t}\sin\theta + \eta_{2,t}\cos\theta \end{bmatrix} \\
&= \begin{bmatrix} p_t\cos\theta - (q_t + \tau p_t)\sin\theta \\ p_t\sin\theta + (q_t + \tau p_t)\cos\theta \end{bmatrix}
\end{aligned}$$

How Sign Restrictions Work

- Accordingly, if we wanted the shocks identified by the Givens rotations to correspond to the true structural shocks, then we need that:

$$\begin{aligned} p_t\left(\cos\theta - \tau\sin\theta\right) - q_t\sin\theta &= q_t + \beta p_t \\ p_t\left(\sin\theta + \tau\cos\theta\right) + q_t\cos\theta &= q_t - \gamma p_t \end{aligned}$$

- This is equivalent to the restrictions:

$$\begin{aligned} \sin\theta &= -1 \\ \cos\theta &= 1 \\ \cos\theta - \tau\sin\theta &= \beta \\ \sin\theta + \tau\cos\theta &= -\gamma \end{aligned}$$

  For arbitrary values of the parameters, these cannot be satisfied.
- Impact effect signs are correct but magnitudes are wrong for *all* the models.

What does it mean for sign restrictions?

- Although sign restrictions-based partial identification is appealing because it requires minimal assumptions, it leads to correspondingly weak conclusions.
- Partially identified models do not appear to present a strong basis for making reliable policy decisions.
- However, they may still be useful *if* you apply the criterion-based median IRF selection method described by Fry and Pagan (2005) to pick a unique, self-consistent model (all of the identified shocks are guaranteed to be orthogonal to each other).

ID through Heteroskedasticity

- Rigobon (2003) describes an alternative to the traditional exclusion restrictions, long-run restrictions, and sign restrictions to identify (or partially identify) an SVAR from a VAR $\rightarrow$ variance/covariance restrictions.
- The intuition is based upon the earliest work on instrumental variables by Wright (1928) and the later work by Rothenberg and Ruud (1990) on simultaneous equation models.
    - If the variance of the innovations for each of the endogenous variables changes over time *and* they do not change in lock-step across endogenous variables, then we can use the variance change to construct a *probabilistic* instrument, which can disentangle the simultaneous effects.
    - For this to work, it requires that the conditional mean be stable over time. It is only the second moments that change.

Example – ID through Heteroskedasticity

- Consider the classic supply-demand system of simultaneous equations with market price and quantity:

$$
\begin{aligned}
p_t &= \beta q_t + \varepsilon_{p,t} \\
q_t &= \gamma p_t + \varepsilon_{q,t}
\end{aligned}
$$

where $\varepsilon_{p,t}$ and $\varepsilon_{q,t}$ are orthogonal. This implies the structural form:

$$
\begin{aligned}
p_t &= \frac{1}{1-\beta\gamma}\left(\beta\varepsilon_{q,t} + \varepsilon_{p,t}\right) \\
q_t &= \frac{1}{1-\beta\gamma}\left(\gamma\varepsilon_{p,t} + \varepsilon_{q,t}\right)
\end{aligned}
$$

- The information available in the data on the two series is summarized in their variance/covariance matrix.

Example – ID through Heteroskedasticity

- The observed variance/covariance matrix of the system is given by:

$$\Omega = \left(\frac{1}{1-\beta\gamma}\right)^2 \begin{bmatrix} \left(\beta^2\sigma_{\varepsilon q}^2 + \sigma_{\varepsilon p}^2\right) & \left(\beta\sigma_{\varepsilon q}^2 + \gamma\sigma_{\varepsilon p}^2\right) \\ \left(\beta\sigma_{\varepsilon q}^2 + \gamma\sigma_{\varepsilon p}^2\right) & \left(\sigma_{\varepsilon q}^2 + \gamma^2\sigma_{\varepsilon p}^2\right) \end{bmatrix}$$

- Notice how there are 4 unknowns $\left(\beta, \gamma, \sigma_{\varepsilon q}^2, \sigma_{\varepsilon p}^2\right)$ and only 3 moment conditions $\left(\sigma_q^2, \sigma_p^2, \sigma_{pq}\right)$ here.
- The system is *unidentified* in the absence of additional information (restrictions).
- Suppose that there are 2 distinct regimes which are distinguished only by their heteroskedasticity. In other words, the variance/covariance matrix is different across regimes, while all the other parameters are unchanged.

Example – ID through Heteroskedasticity

- Then, the variance/covariance matrix for the two regimes is given by:

$$\Omega_s = \left(\frac{1}{1-\beta\gamma}\right)^2 \begin{bmatrix} \left(\beta^2\sigma^2_{\varepsilon q,s} + \sigma^2_{\varepsilon p,s}\right) & \left(\beta\sigma^2_{\varepsilon q} + \gamma\sigma^2_{\varepsilon p,s}\right) \\ \left(\beta\sigma^2_{\varepsilon q,s} + \gamma\sigma^2_{\varepsilon p,s}\right) & \left(\sigma^2_{\varepsilon q} + \gamma^2\sigma^2_{\varepsilon p,s}\right) \end{bmatrix}$$

where $s \in \{1,2\}$.

- Since the conditional means are unchanged, we now have 6 unknowns $\left(\beta, \gamma, \sigma^2_{\varepsilon q,1}, \sigma^2_{\varepsilon q,2}, \sigma^2_{\varepsilon p,1}, \sigma^2_{\varepsilon p,2}\right)$ in 6 moment conditions $\left(\sigma^2_{q,1}, \sigma^2_{q,2}, \sigma^2_{p,1}, \sigma^2_{p,1}, \sigma_{pq,1}, \sigma_{pq,2}\right)$.
- We have *exactly identified* the structural model, so long as the resulting nonlinear equations are independent (similar to the rank condition for linear models). This requires that:
    - β, γ do not change across regimes.
    - ε_q and ε_p are orthogonal.

Example – ID through Heteroskedasticity

- After some algebra, we find that the solution for the slopes must satisfy the system of nonlinear equations:

$$\beta = \frac{\sigma_{pq,s} - \gamma\sigma^2_{p,s}}{\sigma^2_{q,s} - \gamma\sigma_{pq,s}}, s \in \{1, 2\}.$$

- With enough heteroskedastic regimes, the model can be *overidentified*; you can undertake some overidentification tests.
- It is also feasible to accommodate common shocks (and therefore some contemporaneous correlation of the structural shocks), so long as there are not too many common shocks relative to the number of heteroskedastic regimes.
- Rigobon (2003) shows that the procedure is robust to: (1) more heteroskedastic regimes than specified; and (2) misspecification of regime window endpoints.

ID through Heteroskedasticity

- The maintained assumptions are:
  1. No structural breaks in the conditional means (first moments).
  2. Sufficient number of *ex ante* chosen heteroskedastic regimes to generate enough moment conditions for identification.
  3. Set of moment conditions are independent (in a rank condition sense).
- Benefits of this ID strategy:
  - It is good to disentangle simultaneity amongst financial market variables (e.g., asset prices, etc.). These are usually fast-moving and traditional exclusion restrictions are inappropriate.
  - This is an exact identification strategy, unlike sign restrictions.
  - Unlike long-run restrictions, it does not impose any dynamic restrictions (e.g., limited persistence) to be valid. It only uses the contemporaneous variance/covariance matrix for each regime.

Contagion Effects

- Rigobon (2003) estimates the size of contemporaneous linkages across Latin American bond market yields – contagion effects.
- Heteroskedastic regimes are defined by financial market crises – Mexican peso crisis, Asian crisis, etc.
- Endogenous variables are sovereign bond yields for Argentina, Brazil, and Mexico.
- Exogenous variable is the U.S. 10-year government bond yield.
- Common shocks are allowed to enter the error term. Idiosyncratic shocks are assumed to be mutually orthogonal.

Contagion Effects

The structural VAR model is:

$$A \begin{bmatrix} Arg_t \\ Bra_t \\ Mex_t \end{bmatrix} = \alpha + \phi(L) \begin{bmatrix} Arg_t \\ Bra_t \\ Mex_t \end{bmatrix} + \theta US_t + \Theta(L) US_t + \begin{bmatrix} \varepsilon_{Arg,t} \\ \varepsilon_{Bra,t} \\ \varepsilon_{Mex,t} \end{bmatrix} + \Gamma z_t$$

$$\begin{bmatrix} Arg_t \\ Bra_t \\ Mex_t \end{bmatrix} = A^{-1}\alpha + A^{-1}\phi(L) \begin{bmatrix} Arg_t \\ Bra_t \\ Mex_t \end{bmatrix} + A^{-1}\theta US_t + A^{-1}\Theta(L) US_t + \upsilon_t$$

where ϕ and Θ are lag polynomials.

Contagion Effects

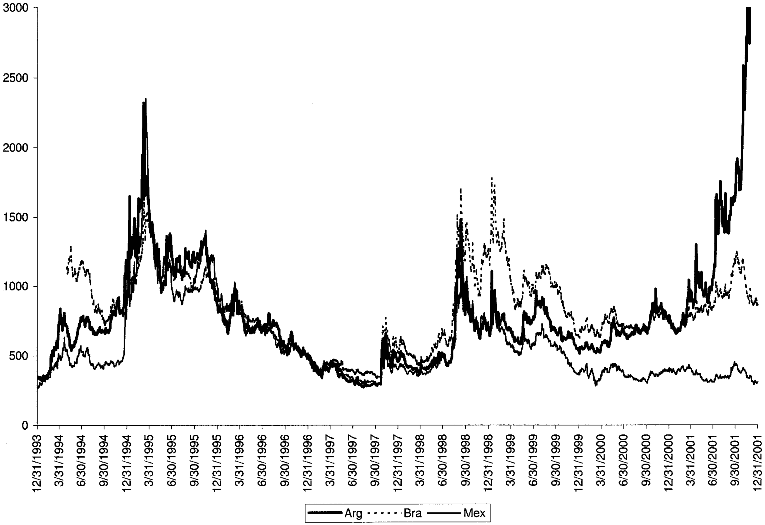
- υ is the reduced form VAR error term, which is estimable.
- It has the following structure under our assumptions:

$$\upsilon_t = A^{-1} \begin{bmatrix} \varepsilon_{Arg,t} \\ \varepsilon_{Bra,t} \\ \varepsilon_{Mex,t} \end{bmatrix} + A^{-1}\Gamma z_t$$

- Thus, the model and the observables can be linked and the parameters identified.

Rigobon (2003) Latin Amer. Bond Yields

FIGURE 2.—YIELDS ON SOVEREIGN DEBT: ARGENTINA, BRAZIL, AND MEXICO



Rigobon (2003) Correlation of Stripped Yields

TABLE 1.—SIMPLE CORRELATIONS OF STRIPPED YIELDS

	Correlation (%)		
Year	Arg–Mex	Arg–Bra	Bra–Mex
1994	82.3		
1995	78.3	78.9	80.4
1996	88.2	90.7	92.7
1997	92.2	94.5	83.1
1998	95.1	94.1	98.7
1999	83.6	73.4	94.2
2000	12.2	67.5	66.7
2001	−37.0	39.5	13.1

Rigobon (2003) Heteroskedastic Regimes

TABLE 2.—TRANQUIL AND CRISIS WINDOWS

Definition of the Windows	Start	End
Tranquil periods	1994-05-01	1994-12-18
	1995-03-02	1997-05-31
	1998-01-01	1998-06-30
	1998-11-01	1999-01-12
	1999-03-01	2000-02-28
	2000-06-01	2000-09-30
Mexican crisis	1994-12-19	1995-03-01
Asian crises	1997-06-01	1998-01-31
Russian crisis	1998-08-01	1998-10-31
Brazilian devaluation	1999-01-13	1999-02-28
Mexico's upgrade	2000-03-01	2000-05-31
Argentinean crisis	2000-10-01	2001-12-31

Rigobon (2003) Contagion Effects

TABLE 4.—POINT ESTIMATES OF CONTEMPORANEOUS COEFFICIENTS

	Arg's Eq.		Bra's Eq.		Mex's Eq.		Common Shock		
Shock	Bra	Mex	Arg	Mex	Arg	Bra	Arg	Bra	Mex
MA	0.0513	0.4071	0.1764	0.2297	0.0965	−0.3949	1.00	−1.0273	−1.8795
	0.1890	0.0895	0.1312	0.0579	0.1892	0.3022		0.3428	0.6529
	0.27	4.55	1.34	3.97	0.51	−1.31		−3.00	−2.88
MAR	0.0298	0.4105	0.3957	0.1530	0.3623	0.0169	1.00	−0.6622	−0.6370
	0.1535	0.0647	0.1443	0.0646	0.1018	0.1227		0.2250	0.2377
	0.19	6.35	2.74	2.37	3.56	0.14		−2.94	−2.68
MARB	0.3674	0.2954	0.2711	0.2147	0.7011	0.1108	1.00	−0.7676	0.5246
	0.3230	0.1342	0.4740	0.2814	0.6641	0.6914		0.9263	1.2044
	1.14	2.20	0.57	0.76	1.06	0.16		−0.83	0.44
MARBU	0.2792	0.3257	0.2044	0.2436	0.5944	0.2111	1.00	−0.8824	0.2891
	0.3977	0.1571	0.4825	0.2438	0.6040	0.6471		0.8033	1.1509
	0.70	2.07	0.42	1.00	0.98	0.33		−1.10	0.25
MARBUA	0.3609	0.3117	0.2310	0.2221	0.0889	0.1443	1.00	−0.9499	−1.2591
	0.1900	0.0761	0.0447	0.0327	0.0653	0.1607		0.3279	0.4167
	1.90	4.10	5.17	6.79	1.36	0.90		−2.90	−3.02

Standard deviations obtained from bootstrapping (500 replications).

Contagion Effects

- Rigobon (2003) finds that:
    - Argentina has a significant response to Mexico, with a doubling of Mexican yields leading to a 30% rise in Argentine yields.
    - Brazil has a significant response to *both* Mexico and Argentina, with a doubling of yields in those respective market leading to approximately 20% rise in Brazilian yields.
    - Mexico has no significant response to *either* Argentina or Brazil.
    - Argentina responds positively to common shocks, while Brazil and Mexico respond negatively to common shocks. Is this "flight-to-quality" in action?
- Quick addendum – standard errors are bootstrapped.

Mapping from models to VARs

- Under some assumptions (not surprisingly), it is comparatively easy to link a dynamic stochastic general equilibrium (DSGE) model to a VAR specification.
  - Why do this? Dynamic responses can be estimated from the VAR which can then be compared to the dynamic responses from a calibrated model, to see how well it captures the empirical properties (at least of the VAR).
  - Christiano, Eichenbaum, and Evans (2005) advocate using the VAR-based impulse responses and a minimum distance criterion to the model-based impulse responses to actually calibrate the DSGE parameters.
  - Note how you could use local projections to generate IRFs which are possibly richer and more robust.

Mapping from models to VARs

- How to proceed?
  1. Linearize (or log-linearize) the first order conditions, etc. of the model.
  2. How to deal with expectations?
     1. Solve forwards and impose some conditions on the underlying innovations (e.g., mean zero), so that a closed-form expression for the expectations in terms of the innovations can be derived.
     2. Think of lags as instruments for expectations.
  3. Result: A VAR or VARMA specification. The model may also give some identifying restrictions. For example, nominal rigidities could constrain the response of prices.
- NB: If the VMA or VARMA representation is non-invertible (no convergent AR representation), it is better to go with a direct MLE procedure instead of a VAR.

Benchmark 3-Equation New Keynesian Model

- Forward-looking representative consumer, with labour supply.
- There is no capital or investment. Production is solely a function of labour input.
- Forward-looking, profit-maximizing, monopolistically competitive firms. Their decisions are made subject to Calvo pricing.
- Monetary policy is given by a Taylor rule for the nominal interest rate.
- Economy is closed; there are no exports/imports.
- There is no government spending or taxation.
- Output is demand-determined, so long as the goods market price is higher than the marginal cost.
- This is the model described by Clarida, Gali, and Gertler (1999) and derived by Bernanke, Gertler, and Gilchrist (1999).

Benchmark 3-Equation New Keynesian Model

- The resulting DSGE model can be specified (after linearizing, including additive innovations, etc.) by 3 equations:
  - An AD curve from the consumption Euler equation with market clearing. This related the output gap to the real interest rate (nominal interest rate minus expected inflation rate) and expected future output gap.
  - A Phillips curve, relating the inflation rate to the current output gap and expected future inflation rate.
  - A Taylor rule, relating the nominal interest rate to the current output gap and current inflation rate.
- Thus, 3 endogenous variables:
  - output gap
  - inflation rate
  - nominal interest rate
- After simplifying, a 3-variable VAR, such as Jorda (2005).

Summary

- There are many options for identification with the VAR framework. This is still a vigorous area of research.
- The choice of identification approach should be a function of the particular research question.
    - Are there natural restrictions based on adjustment costs or rigidities?
    - Simultaneity is problematic for fast-moving variables, like asset prices. Why? The interdependencies mean many ID strategies are just not credible.
    - If the interest is in dynamic responses, you may not need to use a VAR – local projections alternative.
- It is worth thinking about how identification can come from outside the set of variables of interest, as in more traditional instrumental variable methods.